SMOOTHING OF A STATIONARY RANDOM SIGNAL IN A CONTINUOUS MIXER*

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The paper presents results of the first stage of experimental verification of the theoretical concept published in the two first preceding communications concerning the flow of liquid in a continuous nonideal mixer and, namely, the smoothing effect of such a mixer on a passing stationary random concentration signal. In a sufficiently wide range of experimental conditions it has been proven that stochastic fluctuations of the distribution of the residence time of liquid in the mixer do not contribute significantly to the variance of the outlet random signal of the mixer. Further, a good agreement has been proven between the experimental data and the model relationship for the smoothing effect of the mixer based on the application of the gamma distribution of the residence time.

The two preceding communications[1,2] were devoted to the theoretical analysis of the problem of smoothing of a stationary random (concentration) signal on its passage through a continuous mixer. In the first of the two mentioned communications[1] the effect was studied of the stochastic fluctuations proper of a nonideal continuous mixer (showing us stochastic fluctuations of the distribution function of the residence time of liquid) on the smoothing effect of the mixer. The inlet signal of the mixer was then considered as a stationary random function of time. Under this assumption a relationship was derived for the variance of the outlet signal of the mixer (see Eq. (*42*) in ref.[1]) which shall now be written in a somewhat modified form as

$$\sigma_y^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [\hat{f}(t_1)\hat{f}(t_2)\, R_x(t_1 - t_2) + R_f(t_1, t_2)\, R_x(t_1 - t_2)] \,.\, dt_1\, dt_2 \,. \qquad (1)$$

The first part of the integrand in Eq. (*1*) represents the familiar relation (see *e.g.* refs[3,4]) for the linear dynamic system whose time invariant transfer function in this case is identical with the mean value $\hat{f}(t)$ of the probability density, $f(t)$, of the resi-

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dence time of liquid in the mixer. This part of the integrand expresses the variations of the outlet signal due to the variations of the inlet signal. The second term of the integrand is characterized by the autocorrelation function $R_f(t_1, t_2)$ of the probability density of the residence time, $f(t)$, and expresses the increment of the variance of the outlet signal due to the stochastic fluctuations of the liquid flow within the mixer, or corresponding fluctuations of the probability density of the residence times of liquid in the mixer. Values of both parts of the integral in Eq. (*1*) can be found by numerical calculation provided that values of the statistical estimates of the functions in this equation are known.

In the second communication[2] we assumed a continuous mixer in which stochastic fluctuations of the probability density of the residence time of liquid are not significant. This communication also gives reasons for the choice of a particular distribution for the description of the smoothing process, namely the so-called gamma distribution (see *e.g.* ref.[5]), whose probability density was written in the form

$$f(t) = \frac{\varkappa\beta}{\Gamma(\beta)} \exp\left(-\varkappa\beta t\right) (\varkappa\beta t)^{\beta - 1}, \tag{2}$$

where $\varkappa$ and β are parameters characterizing the mean residence time of liquid in the mixer and the intensity of mixing of liquid in the mixer. In the same paper the inlet signal of the mixer was taken to be a stationary random function of time, eventually with a superimposed harmonic component. The autocorrelation functions of this signal was therefore considered in the general form

$$R_x(t) = \sigma_x^2 \exp\left(-A|t|\right) \cos Bt, \tag{3}$$

where A and B characterize the random and the harmonic component of the signal. With the aid of Eqs (*2*) and (*3*) the following relationship was derived[2] for the smoothing effect of the mixer

$$Q = \frac{\sigma_y^2}{\sigma_x^2} = \frac{2\varkappa\beta\Gamma(\beta + 1/2)}{\sqrt{\pi}\Gamma(1 - \beta)\left[(A + \varkappa\beta)^2 + B^2\right]} \sum_{k=0}^{\infty} \varrho^k \left[(A + \varkappa\beta) \cos k\varphi + B \sin k\varphi\right] \cdot \frac{\Gamma(k + 1 - \beta)}{\Gamma(k + 1 + \beta)}, \tag{4}$$

where the quantities ϱ and φ are defined as

$$\varrho = \left[\frac{(A - \varkappa\beta)^2 + B^2}{(A + \varkappa\beta)^2 + B^2}\right]^{1/2} \tag{5}$$

and

$$\varphi = \begin{cases} \pi, & \text{for} \quad A^2 + B^2 < (\varkappa\beta)^2 \\ 2 \operatorname{arctg} \dfrac{2\varkappa\beta B}{(A^2 + B^2 - \varkappa^2\beta^2) + [(A^2 + B^2 - \varkappa^2\beta^2)^2 + (2\varkappa\beta B)^2]^{1/2}} & \end{cases} \tag{6}$$

for other cases.

The relations (4) through (6) enable computation of the numerical value of the smoothing effect Q of the mixer provided that the parameters $\varkappa$ and β, of the probability density of the residence time of liquid in this mixer, and the parameters A and B, of the autocorrelation function of the random inlet signal of the mixer in question, are known.

The goal of the submitted paper can be divided into three parts:

1. For selected model continuous mixers to determine experimentally, by repeated measurements of the responses to an inlet impulse signal (at constant values of the operational parameters of the mixer), statistical estimates of the mean values $\hat{f}(t)$ of the probability density of the residence time of liquid and the autocorrelation function $\hat{R}_f(t_1, t_2)$ of this density.

2. For identical experimental conditions, as in the previous point, to determine variances of the inlet and outlet signals by measurement of the courses of concrete experimental realisations of the random inlet signal and corresponding response at the outlet from the mixer. To utilize the obtained values of the variances for the calculation of the value Q_{exp} of the smoothing effect of the mixer. Further, to evaluate individual contributions to the integral in Eq. (1) and to determine, by comparison, the importance of the effect of the fluctuations of the mixer proper on its smoothing function.

3. For cases, where the effect of the fluctuations of the mixer proper on its smoothing function turns out to be unimportant, to determine from experimental data values of the parameters $\varkappa$ and β of the gamma distribution (2) and of the parameters A and B of the autocorrelation function (3). Then, with the aid of Eqs (4) through (6) to predict the value of the smoothing effect Q_{teor} of the mixer and to compare this value with the result of experiments specified in point 2, *i.e.* with the value Q_{exp}.

The submitted paper was further motivated by the fact that in the chemical engineering literature we have found merely two papers[6,7] presenting experimental results of measurements of the smoothing effect of a continuous mixer. Moreover, in both of these papers deterministic periodic inlet signals were used and the experiments were carried out with atypical mixers: a bubble mixer with central draft tube in the first case[6] and a jet-agitated mixer in the second case[7].

EXPERIMENTAL

A simplified scheme of the apparatus used for experiments is shown in Fig. 1. The principal part is a mixer 1 (see further) fed by liquid from stabilizing tank 7, *via* a regulating valve 6 and a flowmeter 5. The inlet and the outlet of the mixer were equipped with continuous flow conductometric probes 2, the signals of which were processed by means of a two channel conductometer 3 and recorded by chart recorders 4. A syringe 8 served to create the impulse signal during measurements of the distribution of the residence time of liquid in the mixers. During the measurement of the smoothing effect of the mixers the solution of the tracer was injected into the inlet pipe at point 9 by a pump 10 controlled through a transducer 13 by a programmable calculator 12.

Three model mixers, designated as S1, S2, and S3 were used in the experiments. These mixers are shown schematically in Fig. 2 showing also the designations of the principal dimensions, the values of which are given in Table I. All three mixers were cylindrical vessels with either a conical bottom (mixers S1 and S2) or with a flat bottom (mixer S3). All mixers were equipped with radial baffles. Mixer S2 was parted by a perforated wall to two sections of equal volumes; mixer S3 to four sections. A paddle impeller with six blades inclined under 45 degrees was used in the mixer S1. Two, respectively four, six blade turbine impellers with a separating disc were used in the mixers S2 and S3. The volume of liquid in the mixers was 0·0245 cubic meters.

The inlet signal used in the measurement of the smoothing effect of the model mixers was the so-called random telegraph signal (see *e.g.* refs[3,4]). This signal is a stationary ergodic binary signal with required form of the autocorrelation function according to Eq. (*3*) (with a superimposed periodic component). The signal was generated by injection of the tracer into the inlet stream by means of the pump controlled by a programmable calculator. The algorithm of the program in the calculator enabled in a simple way adjustement of the values of the parameters A and B of the autocorrelation function of the inlet signal of the mixer. The details of the method of generation of the random inlet signal to the mixer and details of the experimental method as a whole may be found in ref.[8].

CALCULATIONS

Always $N_r = 20$ realizations were carried out in the measurement of the responses of the used mixers to an impulse inlet signal under identical experimental conditions. The responses of the mixers were recorded on chart recorders. The charts were then read off with a variable time step in order to find all random fluctuations of the responses in the region of very short residence times of liquid without having to evaluate an excessive number of data in that region of the responses where their course is already "smooth". For each series of experiments we thus obtained (after normalizing the records) N_r sequences $f_i(t_k)$, $i = 1, \ldots N_r$ and $k = 1, \ldots N$ of responses at the time instants t_k. In these time instants the mean value of the probability density of the residence time of liquid was determined in the usual way

$$\bar{f}(t_k) = \frac{1}{N_r} \sum_{i=1}^{N_r} f_i(t_k) \tag{7}$$

together with the estimate of the autocorrelation function of the probability density

TABLE I
Principal dimensions of used mixers

Dimension m	Mixer		
	S1	S2	S3
d	0·145	0·098	0·094
D	0·290	0·290	0·290
h	0·075	0·075	0·094
H	0·415	0·415	0·391

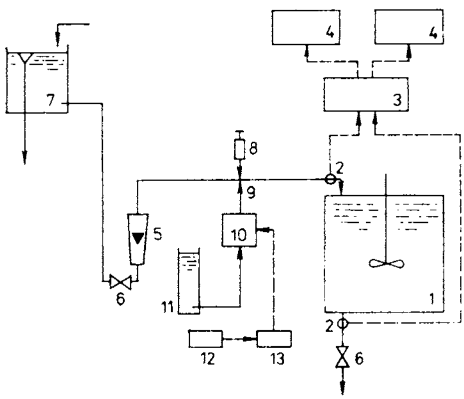

FIG. 1
Scheme of used experimental set-up: 1 mixer, 2 conductivity cell, 3 two-channell conductometer, 4 line recorder, 5 rotameter, 6 regulating valves, 7 storage tank with overflow, 8 injection syringe, 9 injection point of tracer, 10 dosing pump, 11 storage tank for tracer solution, 12 calculator, 13 transducer

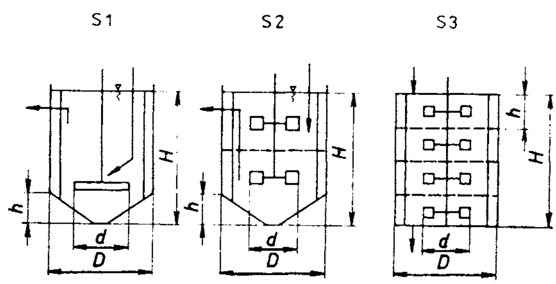

FIG. 2
Principal dimensions of used mixers

of the residence time

$$\hat{R}_f(t_1, t_2) = \frac{\sum_{i=1}^{N_r} [f_i(t_1) - \hat{f}(t_1)] [f_i(t_2) - \hat{f}(t_2)]}{(N_r - 1)}, \tag{8}$$

where t_1 and t_2 assume values from a discrete set t_k, $k = 1, \ldots N$. For $t_1 = t_2$ the estimate according to the last equation represents the estimate $s_f^2(t_k)$ of the variance of the probability density of the residence times.

In the measurements of the smoothing effect of the mixers always only one realization of the experiment was carried out under the given experimental conditions in view of the ergodic property of the inlet signal. The inlet and outlet esignals of the mixer were recorded on chart recorders and the record charts were processed in the following way: N_v values of the sequences $\{x_i = x(iT_v)$ and $y_i = y(iT_v)$, $i = 1, \ldots \ldots, N_v\}$ were read off the charts with a sampling period T_v. These sequences of samples of both signals were used to evaluate the estimates of mean values of the signals

$$\bar{x} = \frac{1}{N_v} \sum_{i=1}^{N_v} x_i, \quad \bar{y} = \frac{1}{N_v} \sum_{i=1}^{N_v} y_i, \tag{9}$$

estimates of the variances of the signals

$$s_x^2 = \frac{1}{N_v - 1} \sum_{i=1}^{N_v} (x_i - \bar{x})^2, \quad s_y^2 = \frac{1}{N_v - 1} \sum_{i=1}^{N_v} (y_i - \bar{y})^2, \tag{10}$$

and estimates of the autocorrelation function of the inlet signal

$$\bar{R}_x(kT_v) = \frac{1}{N_v - k} \sum_{i=1}^{N_v - k} (x_i - \bar{x})(x_{i+k} - \bar{x}). \tag{11}$$

The length of the sampling period T_v and the number of samples N_v were taken so (see *e.g.* ref.[9]) as to keep the relative standard deviations of the estimates of statistical parameters approximately below 15%.

After substituting the estimates $\hat{f}(t_k)$, $\hat{R}_f(t_1, t_2)$ and $\hat{R}_x(kT_v)$ into Eq. (*1*) numerical integration yielded values of individual parts of the integral and their relative magnitude. Further, the value of the variance of the outlet signal, computed with the aid of Eq. (*1*), was compared with the value s_y^2 obtained from the samples of the signal – see Eq. (*10*).

In addition, we obtained by nonlinear regression values of the parameters $\varkappa$ and β of the gamma distribution from the estimates of the mean values $\hat{f}(t_k)$ of the probability density of the residence time for individual series of measurements.

The following weights

$$w(t_k) = \frac{\hat{f}(t_k)}{[\hat{R}_f(t_k, t_k)]^{1/2}} \tag{12}$$

were assigned to individual points of the sequences $\hat{f}(t_k)$ in the computations. Computed values of the parameters $\varkappa$ and β are summarized in Table II.

The statistical estimates $\bar{R}_x(kT_v)$ served to determine (by means of nonlinear regression) the values of the parameters A and B of the autocorrelation function of the inlet signal of the mixer. Examples of the approximation of the experimentally found estimates $\bar{R}_x(kT_v)$ of the function (3) are shown in Fig. 3.

The values of $\varkappa$, β, A, and B, determined by regression, were substituted into Eqs (4)–(6) which in turn served to determine the theoretical value Q_{teor} of the smoothing effect of the mixer which was then compared with the value

$$Q_{exp} = s_y^2/s_x^2 , \tag{13}$$

i.e., the value found by calculation from the experimentally found variances of the inlet and outlet signal. The comparison of Q_{exp} and Q_{teor} is furnished in Fig. 4.

Corresponding sets of experimental data, as well as the details about their processing, may be found in the already cited ref.[8].

A total of 10 series of mesurements of the responses of the used mixers to an impulse signal and 62 experiments aimed at measuring the smoothing effect of the mixer were carried out. These experiments covered the following frequencies of

TABLE II
Values of parameters $\varkappa$ and β

Mixer	n, s^{-1}	$\dot{V}$, m^3 s^{-1}	$\varkappa \cdot 10^3$, s^{-1}	β
S1	—[a]	$1.5 \cdot 10^{-4}$	5·65	1·00
S1	0·833	$1.5 \cdot 10^{-4}$	6·95	1·21
S1	2·500	$1.5 \cdot 10^{-4}$	6·46	1·08
S1	5·000	$1.5 \cdot 10^{-4}$	6·30	1·05
S2	0·833	$1.5 \cdot 10^{-4}$	6·05	1·84
S2	1·666	$1.5 \cdot 10^{-4}$	6·01	1·76
S2	3·333	$1.5 \cdot 10^{-4}$	6·41	1·65
S3	0·833	$1.47 \cdot 10^{-4}$	5·76	3·33
S3	1·666	$1.47 \cdot 10^{-4}$	5·83	3·63
S3	3·333	$1.47 \cdot 10^{-4}$	6·07	3·15

[a] Impeller removed.

revolution of the impellers: 0, 0·83, 2·5, and 5·0 (s^{-1}) for the mixer **S1** and 0·83, 1·67, and 3·33 (s^{-1}) for the mixers **S2** and **S3**. For the mixers **S1** and **S2** we chose the volumetric flow rate of liquid equal $1·5 \cdot 10^{-4}$ ($m^3 s^{-1}$); in view of the higher hydrodynamic resistance we worked in case of the mixer **S3** with the volumetric flow rate $1·47 \cdot 10^{-4}$ ($m^3 s^{-1}$).

The experiments were carried out at laboratory temperature while the temperature was continuously monitored. A correction was implemented[8] on minor temperature variations that eventually occurred during the experiment. Experiments that suffered a major temperature change (and hence also an electric conductivity change) were scratched from the set of experiments.

RESULTS AND DISCUSSION

In view of the large number of experimental data it is not possible to fully present here all of them. Accordingly, we shall confine ourselves to several examples and conclusions following from them. (All experimental data may be found in the cited ref.[8].)

From comparison of the estimate of s_y^2 of the variance of the outlet signal of the mixer, computed from Eq. (*10*) and samples of the signal, with the value of σ_y^2,

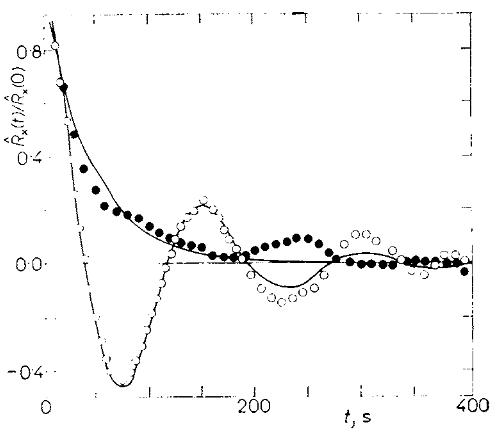

FIG. 3

Autocorrelation function of inlet signal of mixer and its approximation by function (*3*). ○ S1: Impeller removed; $\dot{V} = 1·5 \cdot 10^{-4}$ $m^3 s^{-1}$, ● S2: $n = 5 s^{-1}$; $\dot{V} = 1·5 \cdot 10^{-4}$ $m^3 s^{-1}$ — approximation by function (*3*)

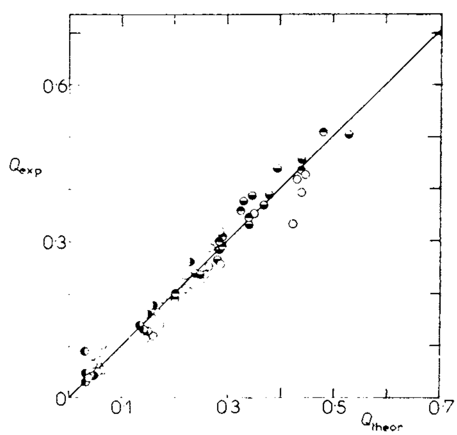

FIG. 4

Comparison of theoretical and experimental values of the smoothing effect. Inlet signal with periodic component S1 ●; S2 ◐; S3 ◑, without peridic component ($B = 0$) S1 ○; S2 ◓; S3 ◒

computed by numerical integration according to Eq. (*1*) using the estimates $\hat{f}(t)$, $\hat{R}_f(t_1, t_2)$, and $\bar{R}_x(t)$, followed for all experiments the mean value of the relative deviation 5·8%. This value may be regarded as good agreement and indicates also a high precision of measurements. From comparison of the values of the first and the second part of the integral in Eq. (*1*) it followed that the second part represented only an insignificant fraction (about 10^{-2} to 10^{-4}) of the first part. The increment of the variance of the outlet signal from the mixer due to the stochastic fluctuations of the residence time distribution was in all cases absolutely negligible in comparison with the "principal" part of this variance corresponding to the random fluctuations of the inlet signal. Fig. 5 shows for illustration the estimate of the variance $s_f^2(t) \equiv \equiv \hat{R}_f(t, t)$ of the probability density in dependence on the residence time and the frequency of revolution of the impellers for the mixer S2. It is apparent that the variance $s_f^2(t)$ assumes more significant values only for very short residence times and hereafter its value very rapidly decreases. From this fact follows the negligible effect of the fluctuations of the mixer proper on its smoothing function. Analogous dependences were found also for mixers S1 and S3.

Fig. 6 shows an example of the agreement between the experimentally found values of $\hat{f}(t)$ of the probability density of the residence time in the mixer S3 and the gamma distribution (*2*). The agreement is very good and it may be stated that analogously good agreement was found also for mixers S1 and S2. An exception to this was only the mixer S1 with the impeller taken out when apparently considerable short-cut liquid flow occurred. This consequently caused considerable deviations of the measured distribution of the residence times from the gamma distribution. From Table II it may be apparent that the value of the parameter beta ranged in the experiments between 1·0 and 3·63 which is a sufficiently wide interval for the assessment of suitability of the gamma distribution (*2*) for description of smoothing mixers. The value of the parameter $\varkappa$ was kept during the experiments at almost constant level and the necessary variability of the variables was achieved by changes of the parameters A and B of the autocorrelation function of the inlet signal of the mixer. Typical courses of the experimentally determined estimates of the autocorrelation function of the inlet signal of the mixer are presented in Fig. 3. The same figure shows also the approximation of these estimates by the function (*3*). The agreement may be regarded as very good which proves the correctness of the selected method of generating the random inlet signal in the form of the random telegraph signal (eventually supplemented by a periodic component). Fig. 4 compares the experimentally determined values Q_{exp} of the smoothing effect of the mixer with the values Q_{teor} computed on the basis of Eqs (*4*)–(*6*) using values of parameters $\varkappa$, β, A, and B determined by regression. For about 70% of the experiments the relative deviation of Q_{exp} and Q_{teor} is less than 10%, *i.e.* the agreement of the experimental and predicted values may be rated as good and sufficient for chemical-engineering needs of description of continuous smoothing mixers. From Fig. 4 it is also apparent that the smoothing of the random signals containing a periodic component is generally better than the smoothing of signals free of this component.

It may be therefore summarized that the experiments briefly presented in this work showed that in the calculations of the smoothing effects of continuous mixers it is well possible to neglect the effect of the fluctuations of the mixer proper. The

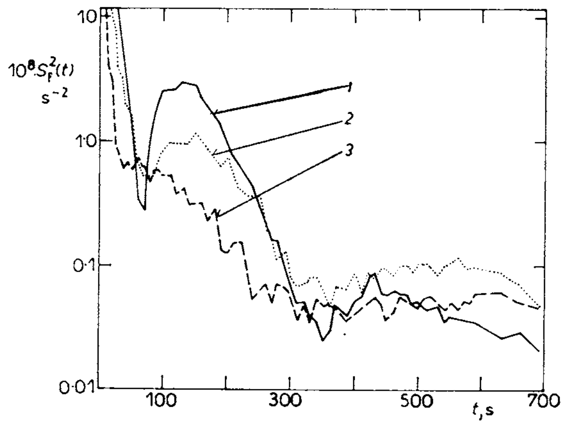

FIG. 5

Dependence of estimate $s_f^2(t)$ of variance of probability density of residence time on the residence time and frequency of revolution of impeller in mixer S2 $\dot{V} = 1 \cdot 5 . 10^{-4}$ m^3 s^{-1}, $n_1 = 3 \cdot 33$ s^{-1}, $n_2 = 1 \cdot 67$ s^{-1}, $n_3 = 0 \cdot 83$ s^{-1})

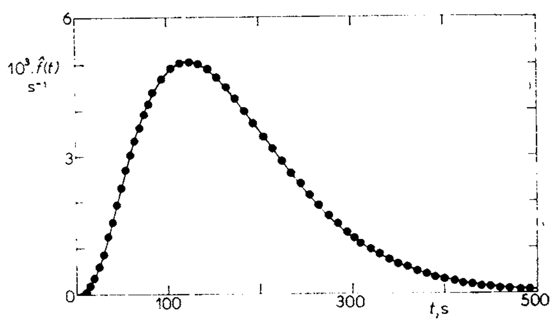

FIG. 6

Experimentally determined probability density of residence time in mixer and its approximation by gamma distribution (*2*) for mixer S2. $n = 1 \cdot 67$ s^{-1}; $\dot{V} = 1 \cdot 5 . 10^{-4}$ m^3 s^{-1}, ● ... experimental values $\hat{f}(t)$; — ... gamma distribution (*2*) mixer may be therefore looked upon as a deterministic linear system. For the modelling of the function of the continuous mixers one can then conveniently utilize Eqs (4)–(6) which simulate with sufficient accuracy the behaviour of real mixers. It is apparent that in special cases the same fidelity of description of the smoothing process could be achieved even with the use of simpler models of the distribution of the residence time than the gamma distribution (2) and from it derived Eqs (4)–(6). The complex form of Eqs (4)–(6), however, is outweighed by their considerable flexibility in the description of real continuous mixers (see ref.[2]). This flexibility is not available if simpler models are used.

It may be expected that the stochastic character of the distribution of the residence time having negligible effect on the smoothing of the random concentration signals could have a marked effect on the course of nonlinear processes (chemical reactions) in continuous mixers. This question deserves a deeper theoretical and experimental study.

LIST OF SYMBOLS

A	parameter of autocorrelation function (3), [T^{-1}]
B	parameter of autocorrelation function (3), [T^{-1}]
$f(t)$	probability density of residence time, [T^{-1}]
i	summation index
k	summation index
n	frequency of revolution of impeller [T^{-1}]
N	number of values
N_r	number of realizations of random function
N_v	number of signal samples
Q	smoothing effect of mixer
$R_z(t_1, t_2)$	autocorrelation function of random function $z(t)$
s^2	statistical estimate of variance
t	time, [T]
T_v	sampling period, [T]
$\dot{V}$	volumetric flow rate [$L^3\ T^{-1}$]
w	statistical weight
$x(t)$	inlet signal of mixer
$y(t)$	outlet signal of mixer
β	parameter of gamma distribution (2)
$\Gamma(z)$	gamma function of z
$\varkappa$	parameter of gamma distribution (2), [T^{-1}]
ϱ	quantity defined by Eq. (5)
φ	quantity defined by Eq. (6)
σ^2	variance

Subscripts

exp	experimental value
f	related to probability density of residence time
i	i-th value of sequence or realization

k	k-th value of sequence or realization
teor	theoretical value
x	related to inlet signal of mixer
y	related to outlet signal of mixer

Subscripts

$\bar{z}$	statistical estimate of mean value of stationary random function $z(t)$
$\hat{z}$	statistical estimate of moments of nonstationary random function $z(t)$

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